16) Let $f(x)=\begin{cases}0, & \text{if } x \text{ is rational}\\ 2x, & \text{if } x \text{ is irrational}\end{cases}$. The surprising property of this function is that it is only continuous at $x=0$. Explain what this means; why this is so; and why f can't be continuous at any other x value.

$$f(x) = \begin{cases} x^2-3x+1 & , x<-1 \\ x+5 & , x \geq -1 \end{cases}$$

only point at which $f(x)$ is not continuous is POSSIBLY $x=-1$ [[joining 2 polynomials]].

a) $f(x)$ is defined at $x=-1$

b) does the 2 sided limit exist?

$\lim_{x \to -1^-} x^2-3x+1 =$

$(-1)^2-3(-1)+1 = 5$

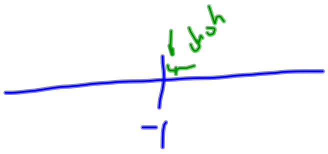
$\lim_{x \to -1^+} x+5 = 4$

2 sided limit does NOT exist

because of the jump discontinuity.

$$f(x)=\begin{cases} x^2-3x+1 & , x<-1 \\ x+5 & , x\geq -1 \end{cases}$$

$$g(x)=\begin{cases} x-3 & , x<2 \\ -x^2+3x-3 & , x\geq 2 \end{cases}$$



$$f+g(x)=\begin{cases} x^2-3x+1+(x-3) & , x<-1 \\ x+5+(x-3) & , -1\leq x<2 \\ -x^2+3x-3+(x+5) & , x\geq 2 \end{cases}$$

Oct 15-7:18 AM

$$f(x)=\begin{cases} x^2-3x+1 & ,x<-1 \\ x+5 & ,x\geq -1 \end{cases}$$

$$g(x)=\begin{cases} x-3 & ,x<2 \\ -x^2+3x-3, & x\geq 2 \end{cases}$$

-1 $\quad$ 2

x	$f(x)$	$g(x)$	$f+g\,(x)$
-2	11	-5	6
-1	4	-4	0
0	5	-3	2
2	7	-1	6
3	8	-3	5
⋮	⋮	⋮	⋮

$f(x)$ is continuous at a point $x=a$ if (and only if):

i) $f(x)$ is defined at $x=a$

ii) $\lim\limits_{x \to a} f(x)$ exists

iii) $\lim\limits_{x \to a} f(x) = f(a)$

$f(x)$ is continuous on an open interval (a,b) if (and only if) $f(x)$ is continuous at every x-value in (a,b)

Polynomials are continuous everywhere.

Rational functions are continuous everywhere they are defined

⋮

14) $\lim_{x \to 9} \frac{12(\sqrt{x}-3)}{x-9} =$

$$\frac{12}{1}\left(\frac{\sqrt{x}-3}{x-9}\right) = \frac{12}{1}\left[\left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)\right] = \frac{12}{1}\left[\frac{\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x}+3)}\right]$$

$$= \frac{12}{1}\left(\frac{1}{\sqrt{x}+3}\right) = \frac{12}{\sqrt{x}+3} = \frac{12}{\sqrt{9}+3} = \frac{12}{6} = 2$$

Oct 15-7:18 AM

#4

$f(x) = \frac{x}{x}$

Set denom. = 0

$x = 0$ this is BAD

so domain is all real numbers except $x = 0$

$$g(x) = \begin{cases} 1, & x < 0 \\ 1, & x > 0 \\ \text{not defined}, & x = 0 \end{cases}$$

$$f(x) = \frac{x^2 - x}{2x}$$

interested in $\lim\limits_{x \to 0} \frac{x^2 - x}{2x} = \lim\limits_{x \to 0} \left(\frac{\cancel{x}(x-1)}{\cancel{x}(2)}\right)$

$$= \lim_{x \to 0} \frac{x-1}{2} = -\frac{1}{2}$$